Some Practical Strategies for Reducing Intermodulation in Satellite Communications

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Various heuristic procedures for obtaining practical solutions to the general one-level carrier frequency assignment problem are described. The problem treated is general in the sense that it accommodates the case where L of the N slots may be explicitly designated as prohibited and unavailable for assignment. This problem occurs in satellite transmission with many small carriers accessing the same transponder where due to multipath and TV interference from crosspolarized transponders of the same satellite and from copolarized transponders of the adjacent satellites, some portions of the bandwidth of the considered transponder cannot be used. To permit comparison with respect to intermodulation (IM)-advantage and central processing unit (CPU) time required, the case without prohibited slots is considered. The sequential insertion procedure in which, starting with two carriers at the two end slots, one additional carrier is optimally inserted at a time to one of the unassigned slots is found best when the ratio between the available bandwidth and the total carrier bandwidth is greater than about 125 percent. All the heuristic procedures produce assignments whose IM-advantages are all greater than the bandwidth ratio.

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INTRODUCTION

When two or more carriers simultaneously access a satellite power amplifier such as a travelling-wave tube amplifier (TWTA) or a solid state power amplifier (SSPA) at the same time, intermodulation products are generated. These products are considered as noise (or interference) which affect the information sent along with the carriers. To ensure that the quality of information received meets a specified requirement in a cost-effective manner, the intermodulation effect must be minimized and subsequently accounted for in the link budget calculation.

The intermodulation effect can be alleviated by reducing the power levels and/or the number of intermodulation products falling into the carrier bandwidths. Since intermodulation product power levels depend on only the carrier input power levels and the amplifier characteristic, the former can be achieved through backoff operation or linearization. The latter can be carried out through careful carrier frequency assignment, since the center frequencies (locations) of intermodulation products depend solely on the center frequencies of the carriers constituting them.

In the backoff operation, the amplifier is operated substantially below its saturated (available) RF output power level, at the backoff region, where the amplifier characteristic is more "linear" to produce low intermodulation product power levels. For satellite power amplifiers, this mode of operation is common even though it wastes most of the amplifier available RF power which is fixed and expensive. Fig. 1 shows, for a typical *Ku*-band satellite TWTA, the relationship between the backoff level and the carrier-to-intermodulation ratio (C/IM) corresponding to the worst center slot (when all the carriers are identical and equally spaced in frequency). The amplifier is typically operated at 4 to 6 dB OPBO (dB backoff from single carrier output saturation level) to yield a C/IM value of 16.5 to 19.5 dB.

Linearization [1–5] requires the use of additional (passive and active) hardware to alter (i.e., "linearize") the amplifier characteristic so that intermodulation product power levels are reduced. Linearization may be acceptable for use with onground power amplifiers [5]. However, in the context of satellite power amplifiers, it may not be acceptable as extra hardware implies extra weight and extra dc power which are both very costly. It also implies a reduction in reliability which is a particularly critical factor when servicing is infeasible.

Carrier frequency assignment has been investigated by many authors [6–16]. In the problem considered, there are K carriers (all having the same bandwidth) accessing the amplifier. The amplifier bandwidth is partitioned into N slots (channels) with each slot having a bandwidth given by the carrier bandwidth plus a guard band on each side. Of the N slots, K must be assigned to the carriers.

Babcock [6], Fang and Sandrin [7], and Atkinson *et al.* [8] investigated the so-called intermodulation-free

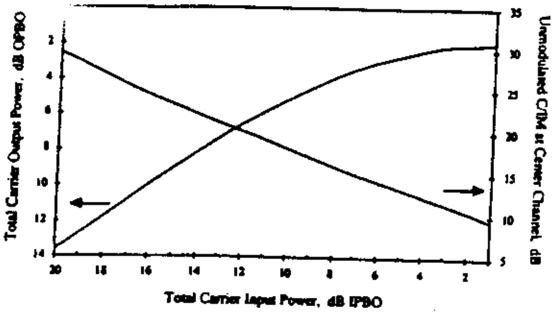

Fig. 1. Typical *Ku*-band satellite TWTA characteristics. Multicarrier operation.

problem where the objective is to ensure that no (third-order) intermodulation products fall into slots assigned to the carriers, using the minimum number of slots N. Optimal assignments for $K \leq 11$ and suboptimal assignments (where N may not be minimum) for $12 \leq K \leq 100$ were given. Edwards *et al.* [9] studied the same problem but with the additional constraint that no assigned slot is allowed to be immediately adjacent to another. This added constraint is useful for avoiding adjacent channel interference in the event that the bandwidth of a slot is not set sufficiently wide. The intermodulation-free problem is usually not of interest, as it requires excessive bandwidth from the amplifiers. For example, for a ten carriers access ($K = 10$), the bandwidth requirement is $N = 62$ and for $K = 100$, $N = 8832$.

When the number of carriers K and the number of slots N are fixed, we note the work of Okinaka *et al.* [10] who provided two general rules to assign frequencies for the multiple-level problem where the carriers are at different power levels and the work of Vuong and Forsey [11] who provided optimal assignments and results for the two-level problem where the amplifier bandwidth is fully utilized ($K = N$).

For the one-level problem, bounds on the number of affecting third-order intermodulation products have been provided by Maxemchuk and Schiff [12] and McClure [13]. Hirata [14] has also provided a bound, however in his analysis ($A + B - C$) intermodulation products are taken to be twice (instead of four times) as powerful as ($2A - B$) intermodulation products; his results therefore are not of practical value.

In the one-level problem, the number of possible assignments of K carriers into N slots is given by $A(K, N) = (N - 2)!/((K - 2)!(N - K)!)$ and the number of ($A + B - C$) intermodulation products is $P(K) = K(K - 1)(K - 2)/2$. When K is small, the problem can be solved using simple exhaustive search procedures in which all possible assignments are examined. However with increasing K, this approach rapidly becomes infeasible; e.g., $A(20, 40) = 3.3578 \times 10^{10}$ (with $P(20) = 3420$) and $A(50, 100) = 2.4968 \times 10^{28}$ (with $P(50) = 58800$). To the authors' knowledge, there is no tractable procedure which provides an optimal solution to the problem; consequently, suboptimal procedures based on rules of thumb and heuristic procedures must be used. Johannsen and Paulsen [15] and Morita *et al.* [16] have suggested the use of assignments that follow the logarithmic and cosine distributions where the carriers are placed more densely near the frequency band edges; however no procedures were given. Okinaka *et al.* [10] have proposed an algorithm which they claimed to be practical (fast in execution time) and produces assignments of reasonably good quality ("quasioptimal" assignments).

Recently, Forbes and Sampaio-Neto [17] derived an algorithm that performs indirect counting and sorting of the third-order intermodulation products generated. It significantly reduces computer time over the conventional direct counting and sorting method when the number of slots N is large ($N > 16$). In an example with the one-level problem and $N = 1366$, the CPU time was reduced from 466 min to 4.3 s. The procedure is based on the use of the fast Fourier transform and, in addition, requires a routine for solving a set of linear algebraic equations (for the two-level or higher level problem).

Several heuristic procedures for solving the general one-level carrier frequency assignment problem are described and evaluated. The problem treated is general in the sense that it accommodates the case where L (≥ 0) of the N slots may be explicitly designated as prohibited and unavailable for assignment. This problem occurs in satellite transmission with many carriers accessing the same transponder where due to multipath and TV interference from crosspolarized transponders of the same satellite and from copolarized transponders of the adjacent satellites, some portions of the bandwidth of the considered transponder cannot be used [18, 19]. The work outlined in this presentation is based on a study carried out by Ozmizrak [23].

STATEMENT OF THE PROBLEM

Like many previous investigations of the problem, the work makes two general assumptions; namely, 1) the carriers can be treated as being unmodulated, and 2) consideration of only ($A + B - C$) intermodulation products provides a sufficiently practical solution. The basis for the first assumption is that most power of a modulated intermodulation product (i.e., intermodulation product that is generated by modulated carriers) falls into the same slot as the product itself. The second assumption is based on the fact that the power associated with ($A + B - C$) products is substantially greater than that of ($2A - B$) third-order products and products of higher order and that there are substantially more ($A + B - C$) products than ($2A - B$) products ($K(K - 1)(K - 2)/2$ as opposed to $K(K - 1)$).

The criterion for evaluating the quality of an assignment is taken to be the maximum number of ($A + B - C$) products falling into an assigned slot. The lower the number, the better is the assignment. This approach is consistent with practical considerations and with previous investigations of the problem, as the worst, not the average nor the total, is a limiting factor in general design.

Without loss of generality, the N slots are numbered 1 through N. Furthermore it can be assumed that neither of the endpoint slots (1 and N) is prohibited because if this was not the case then such a prohibited slot could simply be deleted and the remaining slots renumbered. The set of L prohibited slots, P_L, therefore corresponds to a subset of the integers in the range 2 through $(N - 1)$, with $L \leq (N - K)$.

A feasible assignment g (or simply, an assignment) for the general one-level carrier frequency assignment problem, i.e., the (K, N, P_L) problem, is a set of K distinct values chosen from the set of integer $\{1, 2, \ldots, N\}$ with the restrictions that 1) the values 1 and N are always chosen and 2) no value in P_L is chosen. If $f_r \epsilon g$ then f_r represents the slot assigned to carrier r.

The quality of an assignment, g, is denoted by $Q(g)$. If an assignment g^* is such that $Q(g^*) \leq Q(g)$ for all possible feasible assignments g, then g^* is an optimal solution to the (K, N, P_L) problem. The quality measure $Q(g)$ is defined as follows:

$$Q(g) = \max_r I(f_r)$$

where $I(f_r)$ is the number of intermodulation products falling into slot f_r, i.e.,

$$I(f_r) = \sum_{i=1}^{K-1} \sum_{j=i+1}^{K} \sum_{\substack{k=1 \\ k \neq i \\ k \neq j}}^{K} \Delta(f_i + f_j - f_k, f_r)$$

with

$$\Delta(a, b) = \begin{cases} 1, & \text{for } a = b \\ 0, & \text{otherwise} \end{cases}$$

and f_i, f_j, and f_k being slots assigned to carriers i, j, and k, respectively.

The quality $Q(g)$ of a (K, N, P_L) assignment is often converted to a parameter which is of more direct use to satellite engineers. It is the IM-advantage IMA(g); i.e., the improvement in carrier-to-intermodulation power ratio C/IM over the worst arrangement which occurs when all K carriers are spaced equally over the amplifier bandwidth. This parameter is independent of the characteristic of the amplifier and the power level of the carriers. To find the worst C/IM under the assignment g with a particular amplifier and power level, one just adds this IM-advantage to the corresponding worst C/IM under the worst arrangement which is often given, such as that shown in Fig. 1 for a typical satellite TWTA. The IM-advantage is directly related to $Q(g)$ through the following equation:

$$\text{IMA}(g) = 10 \log_{10}[Q(g_e)/Q(g)] \text{ (dB)}$$

where g_e is a (K, K, Φ) assignment which is clearly unique. Such (K, K, Φ) problems have been extensively studied [22] and in particular, it can be shown that $Q(g_e)$ is the number of $(A + B - C)$ products falling into the center slot(s) and can be computed as follows

$$Q(g_e) = \begin{cases} (3K^2 - 10K + 8)/8, & K \text{ even} \\ (3K^2 - 10K + 9 + 2(-1)^{(K+1)/2})/8, & K \text{ odd} \end{cases}$$

DESCRIPTION OF THE PROCEDURES

The heuristic procedures that are examined in this study are constructed from two primitive operations called deletion and insertion. The deletion operation transforms a (K, N, P_L) assignment into a $(K-1, N, P_L)$ assignment by removing one of the assigned carriers other than those assigned to slots 1 and N. The insertion operation transforms a (K, N, P_L) assignment into a $(K+1, N, P_L)$ assignment by placing one additional carrier into an unassigned but usable slot.

Both these operations are controlled by the two fundamental quantities denoted by $Q(g)$ and $T(g)$. These respectively represent the quality of the assignment g and the total number of intermodulation products falling into K assigned slots associated with the assignment g, i.e.,

$$T(g) = \sum_{r=1}^{K} I(f_r).$$

Figs. 2 and 3 provide the specifications for these procedures. The pseudocode style used in the figures follows the suggestion of Oren [20].

DELINS-INSDEL Procedure

The DELINS-INSDEL procedure is essentially a slight variation of the procedure proposed by Okinaka *et al.* [10]. It is an iterative procedure which generates a new assignment from an old assignment through the successive use of the deletion operation followed by the insertion operation (DELINS) or alternatively, the insertion operation followed by the deletion operation (INSDEL). Its principal improvement over the Okinaka

```
Get a $(K,N,P_L)$ assignment g with slots 1 and N assigned
Repeat (for each carrier within the assignment other than the two carriers
        assigned to slots 1 and N)
    Delete the carrier to obtain a $(K-1,N,P_L)$ assignment $g^-$
    Evaluate and record $Q(g^-)$ and $T(g^-)$
Until (all carriers have been considered)
If (there is a unique $g^-$ for which $Q(g^-)$ is smallest)
    Then  Choose that $g^-$
    Else  If (among the assignments with smallest $Q(g^-)$, there is a
              unique $g^-$ for which $T(g^-)$ is smallest)
              Then  Choose that $g^-$
              Else  Choose the one that was first obtained
          Endif
Endif
```

Fig. 2. Deletion operation.

```
Get a (K. [illegible]) assignment g with slots 1 and N assigned
Repeat (for each usable unassigned slot)
    Insert a carrier into the slot to obtain a $(K+1,N,P_L)$ assignment $g^+$
    Evaluate and record $Q(g^+)$ and $T(g^+)$
Until (all usable unassigned slots have been considered)
If (there is a unique $g^+$ for which $Q(g^+)$ is smallest)
    Then  Choose that $g^+$
    Else  If (among the assignments with smallest $Q(g^+)$, there is a
              unique $g^+$ for which $T(g^+)$ is smallest)
              Then  Choose that $g^+$
              Else  Choose the one that was first obtained
          Endif
Endif
```

Fig. 3. Insertion operation.

et al. procedure is its feature of avoiding a fruitless looping wherein successive iterations produce changing assignments whose quality remains invariant. This is achieved by using the total number of intermodulation products falling into the assigned slots $T(g)$ in both the deletion and insertion operations to deal with multiple assignments that have equal quality. The specification for the DELINS-INSDEL procedure is provided in Fig. 4.

```
Get an initial $(K,N,P_L)$ assignment g
Initialize TEST2 <--- 'success'
Initialize FIRST <--- 'true'
While (TEST2 = 'success') do
    Set $g^+$ <--- g
    Repeat
        Invoke the Deletion operation on $g^+$ to obtain $g^-$
        Invoke the Insertion operation on $g^-$ to obtain $g^+$
    Until (there is no reduction in $Q(g^+)$)
    If ($Q(g^+) = Q(g)$ and $T(g^+) = T(g)$)
        Then  Set TEST1 <--- 'failure'
        Else  Set g <--- $g^+$
              Set TEST1 <--- 'success'
    Endif
    If (TEST1 = 'success' or FIRST = 'true')
        Then  Set $g^-$ <--- g
              Repeat
                  Invoke the Insertion operation on $g^-$ to obtain $g^+$
                  Invoke the Deletion operation on $g^+$ to obtain $g^-$
              Until (there is no reduction in $Q(g^-)$ )
    Endif
    If ($Q(g^-) = Q(g)$ and $T(g^-) = T(g)$)
        Then  Set TEST2 <--- 'failure'
        Else  Set g <--- $g^-$
    Endif
    FIRST <--- 'false'
Endwhile
Accept g as the solution to the $(K,N,P_L)$ problem
```

Fig. 4. DELINS-INSDEL procedure.

SINS and SDEL Procedures

The DELINS-INSDEL procedure can be viewed as a perturbation process which successively refines an initially specified $(K, N, \boldsymbol{P}_L)$ assignment to generate a final suboptimal $(K, N, \boldsymbol{P}_L)$ assignment. An alternate approach that can be considered is to use the greedy principle [21] to construct a $(K, N, \boldsymbol{P}_L)$ assignment. The sequential insertion (SINS) procedure, starting from the $(2, N, \boldsymbol{P}_L)$ assignment with slots 1 and N assigned, inserts one carrier at a time into an unassigned but usable slot in a greedy manner until all K carriers are assigned. The sequential deletion (SDEL) procedure is the counterpart of SINS. It starts from the fully loaded $(N-L, N, \boldsymbol{P}_L)$ assignment where all usable slots are assigned to carriers, and deletes one carrier at a time from its assigned slot (other than assigned slots 1 and N) in a greedy manner until the number of its assigned carriers is reduced to K. The specifications for these two procedures are given in Figs. 5 and 6. Two noteworthy features of SINS and SDEL are: 1) there is no requirement for an initial assignment to start the procedures, i.e., they are self-starting, and 2) a by-product of the solution to the $(K, N, \boldsymbol{P}_L)$ problem is a solution to each $(M, N, \boldsymbol{P}_L)$ problem with $M = 3, 4, \ldots, (K-1)$ for SINS and with $M = (K+1), (K+2), \ldots, (N-L)$ for SDEL.

```
Get K, N and $P_L$
Create the $(2,N,P_L)$ assignment $g_2$ by placing carriers into slots 1 and N
For (each M = 2 to (K-1))
    Create $g_{M+1}$ from $g_M$ using the Insertion operation
Endfor
Accept $g_K$ as the solution to the $(K,N,P_L)$ problem
```

Fig. 5. SINS procedure.

```
Get K, N and $P_L$
Create the fully loaded $(N-L,N,P_L)$ assignment $g_{N-L}$ by placing carriers to
all (N-L) usable slots
For (each M = (N-L) down to (K+1))
    Create $g_{M-1}$ from $g_M$ by using the Deletion operation
Endfor
Accept $g_K$ as the solution to the $(K,N,P_L)$ problem
```

Fig. 6. SDEL procedure.

Other Procedures

Other procedures investigated are either variations or combinations of the abovementioned procedures DELINS-INSDEL, SINS, and SDEL. They include the following.

1) INSDEL-DELINS which is the same as DELINS-INSDEL but the DELINS and INSDEL processes are interchanged;
2) DELINS-INSDEL-J with $J > 1$ which is the same as DELINS-INSDEL, but the deletion and insertion

operation involves J carriers, instead of one carrier at a time. Its counterpart is INSDEL-DELINS-J;

3) DELINS-J with $J \geq 1$ which is the same as DELINS-INSDEL-J but without the INSDEL process. Its counterpart is INSDEL-J;
4) SINS-J with $J > 1$ which is the same as SINS but the sequential insertion process starts with a supplied (J, N, P_L) assignment. Its counterpart is SDEL-J with $J > K$;
5) SINSU which is the same as SINS except that in the insertion operation, the total number of intermodulation products falling into all assigned slots is replaced by the lowest number of intermodulation products falling into an unassigned but usable slot;
6) SINSU-J which is related to SINS-J in a way which is analogous to the relation between SINSU and SINS;
7) DELINS-INSDEL[SINS] which is a procedure where the result from SINS is used as an initial assignment to start DELINS-INSDEL.

COMPUTATIONAL RESULTS

Extensive experimentation has been carried out to evaluate the procedures outlined in the previous section. These experiments used FORTRAN program codes executing on either an Amdahl 470/V8 mainframe or a VAX 11/750 minicomputer system. All timing results quoted are for the latter environment. To save computer time, the Fortes and Sampaio-Neto algorithm [17] was also implemented to perform indirect counting of the number of intermodulation products falling into the slots. Note that unless stated, results shown below are for the case where there are no prohibited slots ($P_L = \Phi$). This permits general comparison among the procedures.

Effect of Initial Assignment on Results with DELINS-INSDEL and Its Variations

The DELINS-INSDEL procedure and its variations are iterative ones which operate on an initially supplied assignment. To study the effect of this initial assignment on the final results and on the execution times required, various patterns for the initial $(K - 2)$ carrier positions have been examined (recall that slots 1 and N must always be assigned). The general conclusions from extensive computer experiments are that 1) the uniform initial assignment where the spacing between the carriers is set equal (or as equal as possible taking into account the constraint imposed by K, N and P_L) is significantly better in terms of CPU time, and 2) the final assignment does vary with respect to the initial assignment but its quality is not very sensitive to the initial assignment. Note that Okinaka *et al.* use uniform initial assignments in generating their reported results with their procedure. All results associated with DELINS-INSDEL and its variations are from uniform initial assignment.

Effect of Including $(2A - B)$ Products

To be more accurate in their results, engineers sometimes take the effect of $(2A - B)$ intermodulation products into consideration where a $(2A - B)$ product is counted as a quarter of an $(A + B - C)$ product to reflect their power level difference [22]. However, it can be easily verified that for a given assignment, the corresponding IM-advantage values computed with and without the inclusion of $(2A - B)$ products are approximately the same (see Table I as an example). In preliminary experiments, it was observed that the explicit inclusion of $(2A - B)$ products increased the CPU time about 10 percent and yielded a final assignment which was different, but nevertheless had approximately the same IM-advantage as the result obtained when only $(A + B - C)$ products were considered. This validates the use of only $(A + B - C)$ products in this research work.

Comparison with Results from Known Procedure and Known Optimal Results

Table I displays some results generated from the three procedures DELINS-INSDEL, SINS, and SDEL together with those supplied by Okinaka *et al.* Note that DELINS-INSDEL is a slight variation of the Okinaka *et al.* procedure which takes both $(A + B - C)$ and $(2A - B)$ products into consideration. From the table it was observed that they produce assignments of practically same qualities.

Table II displays some results generated by the three procedures and the exhaustive search procedure which yields optimal results (additional optimal results can be found in [23]). It was observed that they produce assignments of practical same qualities. This supports the "quasi-optimum" conjecture made by Okinaka *et al.*

Comparison Among the Procedures

Table III displays more results from DELINS-INSDEL, SINS, and SDEL and results from DELINS-1 and DELINS-2. Note that DELINS-1, by definition, is a subset of DELINS-INSDEL and accordingly, it should require less computer CPU time and produces no better assignments than DELINS-INSDEL.

As far as quality is concerned, DELINS-INSDEL is ranked first, with SINS being second and SDEL being last. However, the quality differences are small and for practical purposes these approaches can be considered to be of equivalent quality.

To compare with respect to computer CPU time, one first notes that by using DELINS-1 instead of DELINS-INSDEL the CPU time is cut down but no more than 30 percent in most cases and that DELINS-2 requires more CPU time than DELINS-1. When N is fixed and as K increases, the CPU time associated with SINS increases, that with SDEL decreases, and those with DELINS-2,

TABLE I
Some Results from DELINS-INSDEL, SINS, SDEL, and Okinara et al.

$P_L=\Phi$ (K,N)	Assignment, IM-Advantage (dB) and Execution Time (sec)			
	Okinaka *et al.*	DELINS-INSDEL	SINS	SDEL
(20,40)	1 2 3 5 6 7 10 16 17 19 24 26 27 32 34 36 37 38 39 40	1 2 3 4 5 7 10 12 16 18 19 24 28 29 32 36 37 38 39 40	1 2 3 4 6 8 12 13 18 20 23 26 27 31 34 36 37 38 39 40	1 2 3 4 6 8 11 14 15 19 21 25 26 31 34 35 37 38 39 40
	4.47[1] 4.48[2] ...	4.57 ... 20[3]	4.47 ... 9	4.37 ... 88
(20,60)	1 2 3 6 9 12 13 19 22 27 32 41 43 47 54 55 56 58 59 60	1 2 3 4 7 10 12 17 20 26 32 37 41 44 48 55 56 58 59 60	1 2 4 5 8 13 14 21 22 30 35 37 40 45 49 55 56 58 59 60	1 3 5 8 9 12 15 24 25 27 36 37 45 50 52 54 55 58 59 60
	6.69 6.73 ...	6.69 ... 45	6.53 ... 16	6.53 ... 468
(20,80)	1 2 3 4 8 11 16 28 30 41 45 52 57 61 62 70 73 76 79 80	1 3 4 6 7 13 15 22 25 37 42 52 60 65 69 73 77 78 79 80	1 2 4 8 9 13 21 25 31 35 45 46 59 61 71 72 74 77 79 80	1 2 3 4 7 11 14 15 32 37 47 51 52 60 69 71 72 74 78 80
	8.22 8.18 ...	8.22 ... 65	8.22 ... 22	8.00 ... 919
(30,60)	1 2 3 4 5 6 7 10 13 15 18 22 26 27 32 34 36 40 42 43 47 49 50 54 55 56 57 58 59 60	1 2 3 4 5 7 8 9 10 17 18 23 27 28 32 34 36 40 43 45 46 48 52 53 55 56 57 58 59 60	1 2 3 4 5 7 8 13 14 15 18 21 22 29 30 34 35 37 40 45 47 49 51 53 55 56 57 58 59 60	1 2 3 4 5 7 8 9 11 12 15 22 24 25 27 35 36 37 39 45 50 51 52 53 54 55 57 58 59 60
	4.29 4.33 ...	4.33 ... 160	4.25 ... 59	4.07 ... 440
(30,90)	1 2 3 4 6 9 15 16 19 22 28 31 36 39 42 47 59 60 64 68 70 78 79 81 83 85 86 88 89 90	1 2 3 4 5 6 9 10 16 17 26 29 37 42 45 54 56 60 63 66 72 74 78 80 83 85 87 88 89 90	1 2 4 5 7 8 10 13 19 21 26 31 38 41 45 54 58 59 63 64 72 74 79 80 84 86 87 88 89 90	1 3 4 5 7 10 11 12 13 16 23 31 37 39 40 50 53 54 57 70 71 73 75 78 79 83 85 88 89 90
	6.33 6.37 ...	6.40 ... 361	6.27 ... 104	6.15 ... 2321
(30,120)	1 2 3 4 6 9 13 17 19 32 37 39 48 51 56 62 69 76 77 86 90 96 102 108 110 113 117 118 119 120	1 2 3 5 8 9 13 15 16 23 26 32 41 49 52 58 67 71 83 87 90 95 99 104 114 115 117 118 119 120	1 2 4 8 11 12 13 18 21 31 38 42 45 50 61 63 66 76 81 92 94 96 102 108 114 116 117 118 119 120	1 2 4 5 6 13 15 20 22 29 31 43 46 49 55 69 72 75 76 96 97 106 107 108 111 112 116 118 119 120
	7.88 7.89 ...	7.88 ... 432	7.63 ... 151	7.54 ... 4528

[1] IM-advantage (dB), with $(A + B - C)$ products only.

[2] IM-advantage (dB), with both $(A + B - C)$ and $(2A - B)$ products.

[3] Execution time (sec).

Note: Roman letter symbols here correspond to italic letter symbols in text.

TABLE II
Results from DELINS-INSDEL, SINS, SDEL, and Exhaustive Search Procedure

$P_L=\Phi$	Exhaustive Search		DELINS-INSDEL		SINS		SDEL	
(K,N)	Q(g)	IMA(g)	Q(g)	IMA(g)	Q(g)	IMA(g)	Q(g)	IMA(g)
(5,10)	1	6.02	same*		same		same	
(5,11)	1	6.02	same		same		same	
(6,12)	1	8.45	same		same		2	5.44
(6,17)	1	8.45	same		same		same	
(7,14)	3	5.64	same		same		same	
(7,25)	1	10.41	same		same		same	
(8,16)	4	5.74	same		same		same	
(8,34)	1	11.76	same		same		2	8.76
(9,18)	6	5.23	same		7	4.56	7	4.56
(9,44)	1	13.01	same		same		same	
(10,18)	9	4.61	same		same		10	4.14
(11,18)	13	4.05	same		same		14	3.72
(12,18)	19	3.23	20	3.01	20	3.01	20	3.01
(13,18)	26	2.66	same		28	2.34	27	2.49
(14,18)	35	2.12	37	1.87	36	2.00	36	2.00
(16,19)	55	1.46	same		same		56	1.38
(17,20)	64	1.38	same		65	1.32	same	
(17,21)	59	1.74	60	1.66	60	1.66	same	
(18,21)	74	1.31	same		75	1.25	same	
(18,22)	68	1.67	same		69	1.61	same	
(19,22)	84	1.29	85	1.24	85	1.24	85	1.24
(19,23)	79	1.55	same		same		81	1.45
(20,23)	94	1.23	95	1.18	95	1.18	95	1.18

*Same means that results are identical to those of the exhaustive search procedure.

Note: Roman letter symbols here correspond to italic letter symbols in text.

DELINS-1 and DELINS-INSDEL increase until K is around 70% of N then start to decrease. For cases where the bandwidth ratio (N/K) is large, SINS requires the least CPU time (which is generally less than half of that required by DELINS-INSDEL). When (N/K) is about 125 percent or less, SINS requires the most CPU time and SDEL the least CPU time.

For other procedures, INSDEL-DELINS-J for $J > 1$ or DELINS-INSDEL-J for $J > 1$ in general requires more CPU time and produces slightly worse assignments than DELINS-INSDEL. SINSU, as compared with SINS, produces assignments with about the same quality and requires more CPU time if the direct intermodulation product counting method is used or if the bandwidth ratio $(N/K) > 2$ and the indirect counting Fortes and Sampaio-Neto method is used. SINS-J or SDEL-J should be used in place of SINS or SDEL to reduce CPU time if a good (J, N, Φ) assignment is available. All test runs starting with a limited number of known optimal (J, N, Φ) assignments (which were obtained from the exhaustive search procedure) yielded final (K, N, Φ) assignments which are at least as good as those generated by SINS and SDEL. Finally DELINS-INSDEL[SINS] (or any similar combined procedure), requires CPU time to refine the assignment generated by SINS, but the refinement is often fruitless or insignificant as the assignment is already more or less quasi-optimal.

In summary, by taking both CPU time and IM-advantage into consideration, SINS is recommended, except for cases where K is very close to N ($N/K < 125$ percent), then DELINS-1 should be used.

Bound on IM-Advantages

Fig. 7 is a scattered plot of IM-advantage versus the bandwidth ratio (N/K) in dB for many (K, N) pairs. It

TABLE III
Results from DELIN-2, DELINS-1, DELINS-INSDEL, SINS, and SDEL

$P_L=\Phi$	DELINS-2		DELINS-1		DELINS-INSDEL		SINS		SDEL	
(K,N)	Time(sec)	Q(g)	Time(sec)	Q(g)	Time(sec)	Q(g)	Time(sec)	Q(g)	Time(sec)	Q(g)
(10,40)	3	3	2	3	3	3	2	3	94	3
(15,40)	8	15	5	16	6	16	4	16	92	16
(20,40)	22	44	19	45	20	44	9	45	88	46
(25,40)	34	100	22	101	32	100	18	98	80	99
(30,40)	58	189	53	188	77	187	28	188	62	189
(35,40)	53	324	48	325	66	324	38	325	31	325
(10,60)	4	1	4	1	4	1	2	2	485	2
(15,60)	12	9	11	9	13	9	6	9	485	10
(20,60)	39	28	34	28	45	27	16	28	468	28
(25,60)	62	61	53	61	88	61	32	60	455	63
(30,60)	152	113	135	113	160	113	59	113	440	118
(35,60)	277	191	197	197	241	191	96	188	398	192
(40,60)	312	296	214	299	277	295	142	300	330	306
(45,60)	266	443	162	443	221	442	194	444	262	449
(50,60)	363	638	222	644	391	637	200	640	183	642
(55,60)	225	900	142	899	167	899	213	901	97	900
(10,100)	3	0	3	0	3	0	3	0	3024	0
(20,100)	64	15	68	14	75	14	27	14	3013	14
(30,100)	295	64	251	63	330	62	120	63	3004	65
(40,100)	786	164	769	164	1198	162	340	161	2911	169
(50,100)	1549	342	1447	339	1584	339	726	336	2664	348
(60,100)	1721	616	1501	616	1771	613	1059	611	2229	645
(70,100)	1141	1019	908	1017	1396	1009	1319	1018	1834	1060
(80,100)	765	1593	705	1593	914	1589	1483	1589	1248	1628
(90,100)	1003	2399	921	2401	1009	2401	1615	2406	689	2447

Note: Roman letter symbols here correspond to italic letter symbols in text.

was observed that all the IM-advantage values are greater than their corresponding bandwidth ratio (N/K) in dB.

Results with Prohibited Slots

To illustrate the application of the procedures with prohibited slots, the following example is considered. There are forty 500 kHz carriers accessing a 54 MHz transponder. Due to multipath a 1.5 MHz band at each transponder edge is not used, and due to interference frequency bands (−1, 2) and (−15, −12) MHz relative to transponder center frequency may not be used if its crosspolarized transponder and adjacent satellite copolarized transponder carry FM/TV signals. Table IV displays results of three problems generated from SINS. The first one is without any prohibited slots and its assignment contains three assigned slots that fall into the two frequency bands (slots 24, 26, 55). The second one

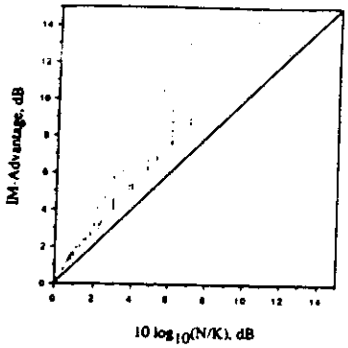

Fig. 7. Scattered plot of IM-advantage versus bandwidth ratio (N/K).

is with prohibited corresponding to the (− 1, 2) MHz frequency band and its assignment contains two assigned slots that fall into the (− 15, − 12) MHz frequency band (slots 24 and 25). The third one is with all prohibited slots. All the three problems here have about the same IM-advantage and the third problem requires the least CPU time.

Note that for all procedures mentioned here, the more prohibited slots the problem has, the less CPU time is required by the problem.

CONCLUSION

The general one-level carrier frequency assignment problem (K, N, P_L) was formulated and various heuristic procedures for solving it have been described and compared in terms of IM-advantage and CPU time. For the (K, N, Φ) problem, SINS is recommended when the bandwidth ratio (N/K) is greater than about 125 percent. For other bandwidth ratios, DELINS-1 or DELINS-INSDEL should be used. Another advantage of using SINS is that its by-products are also solutions to the (M, N, P_L) problem for each M in the range $2 < M < K$. When an optimal or suboptimal (J, N, Φ) assignment for $2 < J < K$ is available, to reduce CPU time, SINS-J should be used in place of SINS. All the heuristic procedures used in this study produce (K, N, Φ) assignments which have IM-advantage greater than the bandwidth ratio (N/K) in dB.

TABLE IV
Results With and Without Prohibited Slots from SINS

(K,N,P_L)	Assignment g	Q(g)	IMA(g)	Time (sec)
(40,102,Φ)	1 2 3 4 5 6 7 8 11 13 17 18 21 24 26 31 40 41 45 49 55 57 59 66 67 71 74 75 79 81 87 90 92 93 96 98 99 100 101 102	158	5.42	345
(40,102,P6)	1 2 3 4 5 6 8 9 11 13 14 21 24 25 31 34 37 41 45 47 49 56 60 66 71 72 75 80 84 85 87 89 93 94 96 98 99 100 101 102	159	5.40	316
(40,102,P12)	1 2 3 4 5 6 8 9 11 13 17 20 21 31 32 34 37 41 45 49 57 59 66 68 71 76 77 79 81 86 87 92 93 95 97 98 99 100 101 102	159	5.40	292

P_6 = {50, 51, 52, 53, 54, 55}
P_{12} = {22, 23, 24, 25, 26, 27, 50, 51, 52, 53, 54, 55}

Note: Roman letter symbols here correspond to italic letter symbols in text.

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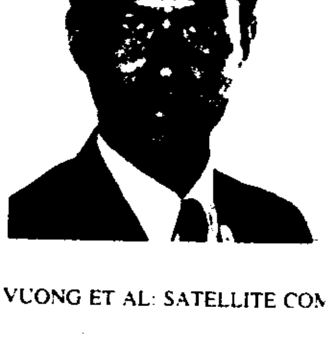

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