

# MATRIX AMPLIFIER AND ROUTING SYSTEM (MARS) ANALYSIS PROGRAM

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*Abstract* Matrix amplifier and routing system (MARS) is a satellite payload technology that can be applied to both commercial and military communication satellites to increase capacity and flexibility. MARS enables the available RF power of satellite high power amplifiers (HPAs) to be automatically shared among channels (transponders or downlink beams). MARS also allows the routing of signals from each uplink beam or channel to one or more of downlink beams without using any switches or multiplexers placed after the HPA outputs. While the MARS concept is simple and straight forward, the MARS operating performance analysis is complex and tedious (in comparison to that of a conventional payload). This paper describes a general-purpose computer program called MARSAP that can be used to analyze performance of a MARS. The emphasis is placed on the mathematical modeling of the MARS components and the algorithms used in the program. General nonlinear/intermod MARS performance results simulated from the program are also presented along with recommendations for future enhancement to the program.

## 1.0 Introduction

Matrix amplifier and routing system (MARS) is a satellite payload technology that can be applied to both commercial and military communication satellites to increase capacity and flexibility. MARS enables the available RF power of satellite high power amplifiers (HPAs) to be automatically shared among channels (transponders or downlink beams). This power sharing avoids typical problems of underloading on one channel (which wastes available RF power) and overloading on another (which degrades bit-error-rate performance, if it is allowed to occur); problems that often occur with conventional satellite payloads. MARS also allows the routing of signals from each uplink beam or channel to one or more of downlink beams without using any switches or multiplexers placed after the HPA outputs, thus reducing satellite payload complexity and satellite payload DC prime power requirements.

The MARS technology is very suitable for the design of satellites that require multiple downlink beams to cover different geographical areas with traffic loads varying significantly with time (e.g., mobile communication satellites).

The first satellite communication systems to implement the MARS technology are: AMSC/TMI's MSAT (first launched in 1995), ETS-6 (launched in 1995), and Inmarsat-3 (first launched in 1996).

While the MARS concept is simple and straight forward, the MARS operating performance analysis is complex and tedious (in comparison to that of a conventional payload). This is because the MARS HPAs are not uniform in practice and are nonlinear with each HPA being accessed by carriers of all channels (instead of just carriers with a channel as in the conventional payload). Also the performance analysis requires

a nonlinearity analysis technique that can estimate accurately not only the power but also the phase of each carrier and each intermodulation product (IMP) generated at each HPA output for coherent combining. Information on each carrier is affected not only by intra-port IMPs (i.e., IMPs created by carriers entering the same MARS input ports) but also by inter-port IMPs (i.e., IMPs created by carriers different MARS input ports) and by carriers entering different MARS input ports (if frequency reuse is employed).

This paper addresses a general-purpose computer program (MARSAP) developed by SAIC to support the Air Force Space and Missile Systems Center to study and to evaluate the MARS technology for possible insertion into future military satellites. The emphasis is placed on the mathematical modeling of the MARS components and the algorithms used in the program. General nonlinear/intermod MARS performance results simulated from the program are also presented along with recommendations for future enhancement to the program.

## 2.0 MARS Description and Mathematical Modeling

As shown in Figure 1, a generic  $M \times K \times N$  MARS consists a set of  $K$  "supposed-to-be-identical" high power amplifiers (HPAs) whose inputs are connected to a linear network called the input matrix and whose outputs are connected to another linear network called the output matrix. The MARS has  $M$  input ports (which are also the input ports of the input matrix) and  $N$  output ports (which are also the output ports of the output matrix). For the MARS to be able to automatically share the HPA power among the ports, the number of the output ports must not exceed the number of the HPAs (i.e.,  $N \leq K$ ) [1, 2].

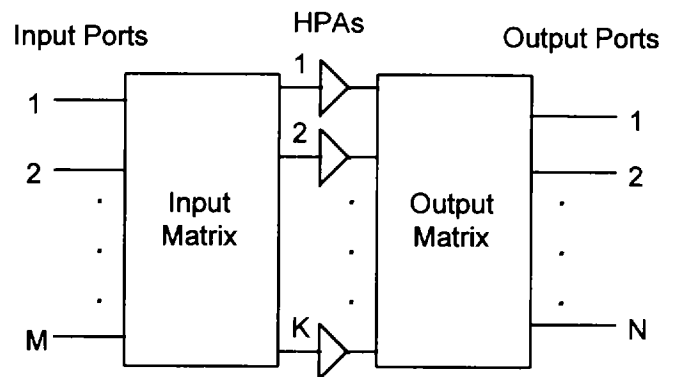


Figure 1. MARS Top Level Diagram

Each signal entering one of the  $M$  input ports is split into  $K$  components of supposedly equal power and appropriate phases set by the input matrix. Each of these components, after amplification, is split further by the output matrix to  $N$  subcomponents of supposedly equal power and appropriate phases set by the output matrix. Consequently, at each MARS

output port, there are a total of KN supposedly equal power subcomponents. These subcomponents are coherent (i.e., originated from the same source) and therefore their voltages are combined together vectorially. The combination can be destructive (totally out-of-phase), constructive (totally in-phase) or somewhere in between (depending on the phases set by the input and output matrices), resulting in the routing of the signal to one or more of the output ports. The mathematical basis and fundamentals of the MARS technology can be found in [1, 2]. Different applications of the MARS technology to replace conventional satellite payloads can be found in [1, 3].

### Input Matrix

For a fixed one-to-one routing MARS, the input matrix can be realized by just only regular hybrids (3-dB, 90° hybrids) [1, 2]. To take advantages of the flexible routing capability of a MARS, the input matrix can be realized with dividers, combiners, and programmable variable phase shifters [1, 2].

Mathematically, the input matrix can be modeled as a linear element (i.e., a linearity) with memory, characterized by a  $K \times M$  matrix transfer function  $\mathbf{P}(f)$ . Each entry  $p_{km}(f)$  of  $\mathbf{P}(f)$  represents a complex transfer function from input port  $m$  to output port  $k$ . In many applications, the dependence on frequency  $f$  is small over the frequency band of interest and can be ignored in the analysis if the same dependence is exhibited for all  $k$  and  $m$ .

Note that for a flexible MARS, the routing vector associated with a MARS input port, say port  $m$ , was introduced in [1] and is defined as the phases of the transfer functions to all the HPAs,

$$\text{Routing Vector (of input port } i) = [\text{phase}(p_{1m}) \text{ phase}(p_{2m}) \dots \text{phase}(p_{km})]$$

The corresponding percentage of power of the signals distributed to the MARS output ports was defined in [1] as the distribution vector.

### High Power Amplifiers (HPAs)

Depending on applications, the HPAs of the MARS can either be traveling wave tube amplifiers (TWTAs) or solid state power amplifiers (SSPAs). The bandwidth of each HPA must be wide enough to accommodate all carriers appearing at all MARS input ports. Each HPA is modeled here as an effectively-memoryless (frequency independent) bandpass nonlinear element (i.e., nonlinearity)<sup>1</sup>. That is, each HPA can be characterized by a frequency-independent, input-envelope-dependent output envelope function  $G(A)$  (the so-called AM/AM function) and by a frequency-independent, input-envelope-dependent phase shift function  $F(A)$  (the so-called AM/PM function). If input to the  $k$ th HPA, characterized by  $G_k(A)$  and  $F_k(A)$ , consists of  $N$  narrowband<sup>2</sup> signals represented as a single composite signal,

$$u(t) = \text{Re}\{A(t)\exp\{j[\omega_0 t + \phi(t)]\}\} \quad (1)$$

then the output to the  $k$ th HPA can be put into the following form:

$$v(t) = \text{Re}\{G_k(A)\exp\{j[\omega_0 t + \phi(t) + F_k(A)]\}\} \quad (2)$$

Note that, in the literature, there are several techniques [5 - 8] that can effectively be used to analyze the loading effects of an effectively-memoryless bandpass nonlinearity. The Fuenzalida-Shimbo-Cook technique [6] (described in Appendix A) was adopted for implementation into the first version of the program, because of its implementation ease and accurate estimation of the powers and particularly the phases of the carriers and intermodulation products at the nonlinearity output. To use the Fuenzalida-Shimbo-Cook technique, the AM/AM and AM/PM functions need to be replaced by a Bessel function series shown below,

$$G(A)\exp\{jF(A)\} \cong \sum_{s=1}^L b_s J_s(\alpha s A) \quad (3)$$

The number of Bessel function terms  $L$  of 10 is often found to be sufficient to represent  $G(A)$  and  $F(A)$ . The complex coefficients  $b_s$ 's and the scaling constant  $\alpha$  can be found by performing curve fittings. Since the curve fitting problem is linear in  $b_s$  and nonlinear in  $\alpha$ , it can most effectively be solved by using the Walling-Lawton-Sylvestre technique [9, 10] which was described in [11] for the specific curve fitting problem defined by Eq. (3).

### Output Matrix

For a fixed one-to-one routing MARS, the output matrix, just like the input matrix, can be realized by just only regular hybrids (3-dB, 90° hybrids) [1, 2]. For a flexible routing MARS, the output matrix characteristics in general are often fixed and realized by regular hybrids just in the case of a fixed routing MARS; and the routing capability is controlled through adjustment of the phases of the input matrix [1, 2].

Mathematically, just like the input matrix, the output matrix can be modeled as a linear element (i.e., a linearity) with memory, characterized by an  $N \times K$  matrix transfer function  $\mathbf{Q}(f)$ . Each entry  $q_{nk}(f)$  of  $\mathbf{Q}(f)$  represents a complex transfer function from input port  $k$  to output port  $n$ . In many applications, the dependence on frequency  $f$  is small over the frequency band of interest and can be ignored in the analysis if the same dependence is exhibited for all  $n$  and  $k$ .

### **3.0 MARS Analysis Algorithm**

The following is a description of the algorithm used in the program MARSAP to analyze the performance of a MARS. The algorithm provides a mechanism to calculate powers of each signal (i.e., carrier or noise) and each intermodulation product at every MARS output port.

1. Define MARS characteristics:

(a) Input matrix characteristics:

$p_{km}(f_j)$  Input matrix transfer function values at frequencies  $f_j$ , for  $k = 1, \dots, K$ ,  $m = 1, \dots, M$  and  $j = 1, \dots, N_p$ . The complex transfer function values can be supplied either as:

<sup>1</sup> See [4] for further description and discussion of effectively memoryless bandpass nonlinearity.

<sup>2</sup> Signals are considered narrowband if their bandwidths are small relative to their frequencies.

- real and imaginary components, or
- amplitude component in dB, power ratio, or voltage ratio and phase component in radians or degrees

- M Number of MARS input ports  
 K Number of HPAs  
 N<sub>p</sub> Number of frequency points associated with the input matrix

(b) HPA characteristics: provided in the form of Bessel function series which have been converted from their basic AM/AM and AM/PM data by performing nonlinear curve fittings discussed earlier,

- b<sub>sk</sub> Complex Bessel coefficients associated with the kth HPA, for s = 1, ..., L<sub>k</sub> and k=1, ..., K  
 α<sub>k</sub> Real scaling constant associated with the kth HPA, for k = 1, ..., K  
 K Number of HPAs  
 L<sub>k</sub> Number of terms in the Bessel function series of the kth HPA  
 P<sub>ri,k</sub> Input power (in dBm) at the reference point (e.g., one-dB compression point, or saturation point) where the AM/AM and AM/PM functions G<sub>k</sub>(A) and F<sub>k</sub>(A) of the kth HPA are normalized to unit envelope, for k = 1, ..., K  
 P<sub>ro,k</sub> Output power (in dBm) at the reference point associated with the kth HPA, for k=1, ..., K

(c) Output matrix characteristics:

- q<sub>nk</sub>(f<sub>j</sub>) Output matrix transfer function values at frequencies f<sub>j</sub>, for k = 1, ..., K, n = 1, ..., N and j = 1, ..., N<sub>q</sub>. The complex transfer function values can be supplied either as:
- real and imaginary components, or
  - amplitude component in dB, power ratio or voltage ratio and phase component in radians or degrees
- N Number of MARS output ports  
 K Number of HPAs  
 N<sub>q</sub> Number of frequency points associated with the output matrix

2. Define the carriers and noise at each MARS input port. Note that in the current version of the program MARSAP, due to the restriction of the Fuenzalida-Shimbo-Cook technique, only one noise source is allowed for the analysis. If more than one noise is desirable, they must be approximated as carriers.

- N<sub>cm</sub> Number of carriers at MARS input port m, for m = 1, ..., M  
 P<sub>cm</sub> Power (in dBm) of carrier c at MARS input port m, for c = 1, ..., N<sub>cm</sub> and m = 1, ..., M  
 f<sub>cm</sub> Center frequency (in MHz) of carrier c at input port m, for c = 1, ..., N<sub>cm</sub> and m = 1, ..., M  
 I<sub>c</sub> Noise indicator, indicating at which MARS input port the noise exists (I<sub>c</sub> = 0, if noise does not exist)  
 P<sub>noise</sub> Noise power (in dBm) at input port I<sub>c</sub>, if noise exists

f<sub>noise</sub> Noise center frequency (in MHz) at port I<sub>c</sub>, if noise exists

M Number of MARS input ports

3. Calculate powers and phases of all signals (carriers and noise) at each HPA input by adjusting the powers and phases of the signals with the input matrix transfer function values.
4. If any of the carriers are coherent, combine their voltages vectorially.
5. Normalize the powers of the signals associated with the kth HPA by the input power at the reference point, P<sub>ri,k</sub>, and convert the normalized powers to envelopes A<sub>c</sub>'s and σ, for all carriers and noise and for k = 1, ..., K.
6. Using the Fuenzalida-Shimbo-Cook technique (described in Appendix A), calculate the complex envelopes of the output signals and of the 3rd order IMPs, and calculate the frequencies of the products. Note that to reduce computer time and storage, only the envelopes and frequencies of the products that fall into the frequency window of interest (supplied by the user) are calculated.
7. Convert the complex envelopes to the normalized powers and phases and then scale the powers using the output powers at the HPA reference points, P<sub>ro,k</sub>, with k = 1, ..., K.
8. At each MARS output port, calculate powers and phases of signals and IMPs by adjusting the powers and phases of signals and IMPs at the outputs of the HPAs with the output matrix transfer function values.
9. At each MARS output port, combine together coherent signals and coherent IMPs.
10. Put the characteristics of the signals and IMPs into proper formats for printout or filing.

#### 4.0 MARS Analysis Program MARSAP

The current version of the program MARSAP was written in Microsoft's structured FORTRAN, Version 5.1, for use on an IBM-386/486 (or its compatible) with WINDOWS software to fully utilize its RAM memory.

Outputs from the program are the powers and frequencies of each signal and each IMP at the specified MARS output ports and within the specified frequency windows. These powers can be used directly to evaluate other MARS performance metrics such as port-to-port isolations, carrier-to-noise ratios (C/Ns), carrier-to-interference ratios (C/Is), and carrier-to-intermodulation ratios (C/IMs).

The program MARSAP provides the capability to model a wide range of potential MARS designs, and includes the following features:

- Variable number of MARS input ports
- Variable number of MARS output ports
- Variable number of HPAs
- Input matrix modeled as a linearity with memory which is characterized by any linear matrix transfer function and can be supplied as a file
- Output matrix modeled as a linearity with memory which is characterized by any linear matrix transfer function and can be supplied as a file

- Each HPA modeled as an effectively memoryless nonlinearity whose AM/AM and AM/PM characteristics can be different and can be supplied (in terms of Bessel function series parameters) as a file
- Carriers having any specified input powers and frequencies
- One input noise source having any specified input power and frequency (optional)

By adjusting the configuration of the inputs, the program can also be used to analyze the performance of a single HPA (e.g., SSPA, TWTA). The program MARSAP can also be used to perform sensitivity analysis due to nonuniformity in magnitudes and phases of the input and output matrices across the frequency band of interest and due to nonuniformity in HPA characteristics (caused during manufacturing processes and environmental temperature change).

### 5.0 MARS Simulation Results

Through simulation using the program MARSAP (and also directly from mathematical analysis, some general characteristics of a MARS were deduced. They include

- General routing characteristics of a MARS, i.e., the routing vectors and their corresponding distribution vectors which were summarized in [1], [2]. The general results are that for  $K=2$ , signals from one input port can be routed to both output ports with any proportion of power split (i.e., any value of the distribution vector). For other values of  $K$  of the form  $K=2^k$  where  $k$  is an integer greater than 1, it is not possible to route signals to obtain any arbitrary values of the distribution vector; however, it is always possible to route signals to any  $2^i$  output ports with equal power split for any  $i=1, 2, \dots, k$ .
- Inter-port IMP characteristics for a general fixed one-to-one routing MARS. In the ideal cases (where there are no errors in the design of HPAs and the input and output matrices), each of the inter-port 3rd order IMPs (i.e., (A+B-C) and (2A-B) where not all carriers A, B and C are not destined to the same output port) appears only at one and only one output port. The inter-port IMP destinations depend on the destinations of the carriers A, B and C. The dependence was obtained and shown in [1, 2]. The power of an IMP was also found to be independent of the destinations of the carriers A, B and C.
- General intermodulation performance of a fixed one-to-one routing MARS versus a conventional payload. The MARS has  $K$  input ports,  $K$  identical HPAs, and  $K$  output ports. The conventional payload, for comparison purposes, has  $K$  identical HPAs. Each input port of the two systems is loaded with equal number ( $Z$ ) of carriers which are equal in power and equally spaced in frequency). The comparison shows i) when frequency reuse among the ports is not employed, the MARS improves IM performance (i.e., MARS has better C/IM associated with the worst carrier (center carrier)) and ii) when frequency band associated with each input port is fully reuse, the performance of the MARS is worse than but approaches that of the conventional payload [1, 2]

### 6.0 Recommendations

Recommendations for future enhancement to the program MARSAP are:

- Development of graphics user interface (GUI) to improve the program's user-friendliness in terms of its input and output displays.
- Inclusion of 5th order (or even higher order) IMPs. While the use of only 3rd order IMPs are sufficiently accurate in almost all applications, in specific cases, such as where the differences in power levels among the carriers are large and the number of carriers involved is small, the inclusion of higher order products may be necessary.
- Calculation of modulated carrier and IM power spectra. The calculation would improve the accuracy of the inband C/I and C/IM values required in the link budget calculation to obtain  $E_b/N_0$  and BER.
- Inclusion of other amplifier loading techniques such as the IM-Microscope (Stochastic Gain) technique [5] to complement the Fuenzalida-Shimbo-Cook technique. While the Fuenzalida-Shimbo-Cook technique provides a very simple way to accurately estimate the output powers and phase shifts of the carriers and IM products, its computing time (and also memory) grows exponentially with respect to the number of carriers involved in the analysis. The computing labor associated with the Stochastic Gain technique does not change very much (less than linear) with respect to the number of carriers involved.
- Extension of the Fuenzalida-Shimbo-Cook technique to handle multiple noise inputs [12]. Presently, the Fuenzalida-Shimbo-Cook technique handles only one noise source and in general, to be accurate in the analysis, each input port should consist of one bandlimited white noise signal (front end composite uplink thermal noise). Also, for analysis with a large number of carriers, to reduce the computer time, some of these carriers can be grouped together and replaced with noise.

### Appendix A: The Fuenzalida-Shimbo-Cook Technique

From the Fuenzalida-Shimbo-Cook technique [6], input to an effectively memoryless bandpass nonlinearity such as a TWTA or SSPA exhibiting both AM/AM characteristic  $G(A)$  and AM/PM characteristic  $F(A)$  is assumed to consist of  $M$  modulated carriers and Gaussian noise,

$$u(t) = \text{Re} \left\{ \sum_{i=1}^M A_i \exp[j2\pi f_i t + j\phi_i(t)] + [N_c(t) + jN_s(t)] \exp[j2\pi f_n t] \right\} \quad (\text{A1})$$

where  $A_i$ ,  $\phi_i(t)$ , and  $f_i$  are, respectively, the real envelope assumed to be constant, phase, and frequency of the carrier  $i$ ; and  $N_c(t)$  and  $N_s(t)$  are the real and imaginary parts of the noise complex envelope centered at frequency  $f_n$  and normally distributed with zero-mean and  $\sigma^2$ -variance,

$$N_c(t) \sim N(0, \sigma^2) \quad (\text{A2a})$$

$$N_s(t) \sim N(0, \sigma^2) \quad (\text{A2b})$$

Through the use of the double Fourier transformation, the output power spectrum can be put into the following form:

$$S(f) = \sum_{\substack{k_1, k_2, \dots, k_{M+1} = -\infty \\ k_1 + k_2 + \dots + k_{M+1} = 1}}^{\infty} (1/2) \sum_{q=0}^{\infty} |N(k_1, k_2, \dots, k_{M+1}; q)|^2 \cdot \Omega(k_1, k_2, \dots, k_{M+1}; q; f - (\sum_{i=1}^M k_i f_i) - k_{M+1} f_N) \quad (A3)$$

where

$$\sum_{q=0}^{\infty} (\cdot) = \sum_{q=0}^{\infty} (\cdot) \quad \text{for } k_{M+1} \neq 0 \quad (A4a)$$

$$= (\cdot) \text{ evaluated at } q=0 \quad \text{for } k_{M+1} = 0 \quad (A4b)$$

$$N(k_1, k_2, \dots, k_{M+1}; q) = \sum_{s=1}^L \{ b_s \exp(-\alpha^2 s^2 \sigma^2 / 2) [\prod_{i=1}^M J_{k_i}(\alpha s A_i)] \cdot \{ (\alpha^2 s^2 \sigma^2 / 2)^{2q + |k_{M+1}|} / [q! (|k_{M+1}| + q)!] \}^{1/2} \} \quad (A5)$$

with L being an integer,  $\alpha$  being a real constant, and  $b_s$  being complex constants which are related to G(A) and F(A) by the following equation for all operating values of composite input envelope A,

$$G(A) \exp[jF(A)] \cong \sum_{s=1}^L b_s J_1(\alpha s A) \quad (A6)$$

and finally,

$$\Omega(k_1, k_2, \dots, k_{M+1}; q; f) = S_{1, k_1} * \dots * S_{M, k_M} * S_{M+1, 2q + |k_{M+1}|} \quad (A7)$$

with

$$S_{i,j} = \delta(f) \quad \text{for } j = 0 \quad (A8a)$$

$$= S_i(f) \quad \text{for } j = 1 \quad (A8b)$$

$$= S_i(f) * S_i(f) * \dots * S_i(f) \quad (j-1) \text{ times for } j > 1 \quad (A8c)$$

$\delta(\cdot)$  being the impulse function, \* being the convolution operator,  $S_i(f)$  with  $i \leq M$  being the low pass equivalent of the power spectrum of carrier I normalized to unit power, and  $S_{M+1}(f)$  being the power spectrum of  $N_c(t)$  or  $N_s(t)$  normalized to unit power.

Note that each term in Eq. (A3) is associated with a carrier, noise, or an intermodulation product, depending on the value of the (M+2)-tuple  $(k_1, k_2, \dots, k_{M+1}; q)$ . For the  $i$ th carrier,  $k_i = 1$ ,  $k_j = 0$  for  $j \neq i$  and  $j = 1, \dots, M+1$ , and  $q = 0$ . For the noise,  $k_{M+1} = 1$ ,  $k_j = 0$  for  $j = 1, \dots, M$ , and  $q = 0$ . And for an intermodulation product, its order and center frequency are determined by the following equations,

$$\text{Order} = \left( \sum_{i=1}^{M+1} |k_i| \right) + 2q \quad (A9)$$

$$\text{Center Frequency} = \left( \sum_{i=1}^M k_i f_i \right) - k_{M+1} f_N \quad (A10)$$

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